Experimental Economics

Lecture 5: Bounded Rationality

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„In my point of view, this paper included extensive economical language making it less accessible to individuals without prior knowledge on the matter. The points that are made are interesting yet can be difficult to grasp upon first lecture. A different writing style would have been more efficient in bringing forwards the points that the authors are trying to make. What I found the most interesting is the propensity to which individuals’ behaviours can change according to the modification of one parameter in a game.“
theoretical Nash-Outcomes if you apply the treasure treatment to these games. This means the games are set up in a way that the behavior conforms nicely to the predictions made by the theory beforehand. On the other hand, they also show that you can apply the contradiction treatment, that means setting up the games in a fashion that results in behavior deviating from the predictions made by theory. Furthermore, they somewhat provoke these contradictory results in order to emphasize the importance of possible deviating behavior of subjects and highlight the area in which current theoretical concepts have weaknesses explaining the behavior taking place. They identify, depending on the game played and the characteristics that come with it, possible reasons for the deviation of behavior from the rational and optimal outcomes predicted by theory. They show that payoff magnitudes play a very important role in the outcome of games. The result shows that the amount of the payoff does not influence Nash-outcomes as long as the relation between the payoffs stays the same, but in reality do observe that the behavior of the subjects does change when the payoff magnitudes change. The authors show plenty of different
The results for the private value sealed-bid auction do not surprise me the slightest. In this scenario the vast majority is just focused on winning the auction, not on making a theoretically optimal decision. Winning the auction and getting hand on at least some pay-out is way more graspable than knowing that you did the right thing and having the best theoretical pay-out although you just lost the auction to that one guy who just bid 6$ on something that’s worth 6$. Being a somewhat decent poker player I encounter players with this mindset over and over but also know by myself how hard it is to overcome sometimes. It would be interesting to ask the players how good they feel after the auction and compare the happiness of those who chose optimal bids with those who won the auction.

In the conclusion the author and I concur. The general public might not understand game theory very well, which leads some experiments, when tweaked right, to differ strongly from the equilibrium results. However, on a basic level, humans tend to understand humans pretty well. This leads not only to the results presented in the paper being rather intuitive for the reader but also in such behavior being anticipated from players really well.
“Game theory is for proving theorems, not playing games” is what the Nobel Prize winner Reinhard Selten shared. Of course, the traditional games are engaging and have been a part of a normative theory when we talk about in what way rational people should play games with each other instead of a positive theory predicting us actual behavior. When it comes to individual decision making it is very normal to separate normative and positive studies that gives us the opportunity to analyze actual and optimal decision making. When it comes to games the normative-based defense is not convincing, the best way for one to play games is depending on how others actually play and not in a way that a theory dictates that rational people should play. John Nash who is also a Nobel winner lost all confidence in relevance of game theory when his experiments did not provide support to theory, there was no way around this dilemma.

Does one-shot really exist? Well yes, there are instances where decisions are made independently and only once in people’s lives; but examples are very rare. For instance, writing a will and testament or the Prime Ministers Letters of last resort. Although these are hardly decided in nonchalant, time constrained, low stakes, isolated laboratory environment. So to infer any generalizability is tenuous.
Memos

As mentioned before, all the anomalous behavior on one-shot game could be explained, economists have theories for it and could calculate everything using computer simulation and other processes. From my perspective, one-shot game means there is no second chance, therefore people tend to be selfish, envious, or feel guilty over this short span because they were just trying to do what they think could maximizes their utility and choose more secure option for them. This really shows their genuine behavior without influence from learning experience that they would have gotten with multi-move game.

One important caveat to note: throughout Goeree and Holt’s paper and prevalent in much of the game theory literature is a discourse that treats what I would term as ‘market rationality’ or ‘economic rationality’ as the only existing rationality, or at least as perfect rationality and decision-making; as if such a thing existed. This implicit assumption in game theory, that Goeree and Holt peddle, that it is rational, and perfect or optimal decision-making to always seek more wealth and for monetary factors to precede all others is neither universal nor widely applicable in reality. It is an important term distinction that has subtlety
Individual $i$ at time $t = 0$ maximizes expected utility subject to a probability distribution $p(s)$ of the states of the world $s \in S$:

$$\max \sum_{t=0}^{\infty} \delta^t \sum_{s \in S_t} p(s_t) U(x_i^t | s_t)$$

$x_i^t \in X_i$

We have already considered the following deviations from the standard model:
- Non-standard preferences (e.g., social preferences)
- Non-standard beliefs (e.g., non-Bayesian updating)
Today: Non-standard decision making
Non-Standard Decision Making

• Heuristics and biases:
  • Heuristic: short-cut to solving decision problem
  • Other examples: framing, nudging
  • System 1 vs System 2 cognition (Kahnemann, 2002)
  • Emotions

• This week: bounded rationality
  • Even when our judgment is not clouded by our biology (e.g., heuristics), we may still not act fully rationally
  • This lecture: focus on games/strategic reasoning
Bounded Rationality

- Sometimes making the optimal decision is impossible
Bounded Rationality

• We know from the proof by John Nash on the existence of equilibria in finite games that Chess and Go have an equilibrium.
• But no person/computer has been able to calculate it.
Bounded Rationality

• For firms selling natural resources (oil, coal, wine?), life can be very complicated:

\[
\max J_i = \int_0^\infty \left\{ \{\bar{p} - (n - 1)\mu_1 x_i - [\mu_1(n - 2) + \mu_2]z - q_i\}q_i 
- q_i(\bar{p} - cx_i)\right\} e^{-\delta t} dt
\]

s.t. \[ \dot{x}_i = -q_i, \quad x_i(0) = x_{i0} > 0, \quad \lim_{t \to \infty} x_i \geq 0 \quad \forall i \in \{1, \ldots, n\}, \]

\[ \dot{z} = -(n - 1)\mu_1 x_i - [\mu_1(n - 2) + \mu_2]z, \quad z(0) = \sum_{j \neq i} x_{j0}. \]

• Salo and Tahvonen (2001) can solve this problem, but most people probably cannot
Bounded Rationality

• For firms selling natural resources (oil, coal, wine?), life can be very complicated:

Proposition 3. Eqs. (20) and (21) possess a unique solution, $\mu_1 < 0$, $\mu_2 > 0$, so that $r_1$ and $r_2$ are real and negative. This solution must lie in the cone $\mu_1 < 0$, $\mu_2 + (n - 1)\mu_1 > 0$ and is given by $\mu_1 = -a_1/\sqrt{1 + s^2}$, $\mu_2 = [a_2 s + \frac{1}{2} a_1 (n - 1)]/\sqrt{1 + s^2} - \frac{1}{2} \delta$, where $s = a_0/3\{p_0 + 2p_1 \cos[\frac{1}{3} \arccos(p_2/p_3)]\}$, $p_0 = -1 + \gamma + 3n + \gamma n$, $p_1 = (16 + 4\gamma + \gamma^2 + 24n + 16\gamma n + 2\gamma^2 n + 6\gamma n^2 + \gamma^2 n^2)^{1/2}$, $p_2 = 44 + 30\gamma + 6\gamma^2 + \gamma^3 + (180 + 102\gamma + 30\gamma^2 + 3\gamma^3)n + (108\gamma + 33\gamma^2 + 3\gamma^3)n^2 + (9\gamma^2 + \gamma^3)n^3$, $a_0 = 1/\sqrt{(3n + 1)(n - 1)}$, $a_1 = a_0 \sqrt{\delta(2c + \delta)}$, $a_2 = \frac{1}{2} \sqrt{\delta(2c + \delta)}$ and $\gamma = \delta/c$ (Proof: Appendix A).

• Salo and Tahvonen (2001) can solve this problem, but most people probably cannot.
Bounded Rationality

• Rational agents always make the optimal (rational) decision

• Real people may not be able to make the optimal decision
  • If no one/only experts can compute it
  • If it takes too much time/effort to compute
  • If we are inattentive/make mistakes
  • Etc.

• If people are as smart as rational agents...
  • Then why do you need to be taught how to solve models?
But if not optimal, then what?

- Problem (for economists): there is only one way to be smart, but there are infinitely many ways to be stupid (irrational).

- Behavioral economics attempts to identify robust, widespread patterns of irrationality and then create models incorporating these irrationalities.

- The goal of behavioral models:
  - Predict/explain behavior better than rational model
  - In a large number of settings
  - In a parsimonious way (not just extra parameters)
  - Behaviorally realistic.
Simple games
(Beard and Beil 1994)

\[ \begin{array}{c}
1,2
\downarrow
L \\
\uparrow
1 & r
L \\
\downarrow
9.75; 3 & 9.75, 3
R \\
\uparrow
3; 4.75 & 10; 5
\end{array} \]

• Sequential game: Player 1 moves first
• How much does player 1 rely on the rationality of player 2?
Treatments
(Beard and Beil, 1994)

Payoffs

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Player A Chooses L</th>
<th>Player B Chooses L</th>
<th>Player B Chooses r</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(9.75, 3.00)</td>
<td>(3.00, 4.75)</td>
<td>(10.00, 5.00)</td>
</tr>
<tr>
<td>2</td>
<td>(9.00, 3.00)</td>
<td>(3.00, 4.75)</td>
<td>(10.00, 5.00)</td>
</tr>
<tr>
<td>3</td>
<td>(7.00, 3.00)</td>
<td>(3.00, 4.75)</td>
<td>(10.00, 5.00)</td>
</tr>
<tr>
<td>4</td>
<td>(9.75, 3.00)</td>
<td>(3.00, 3.00)</td>
<td>(10.00, 5.00)</td>
</tr>
<tr>
<td>5</td>
<td>(9.75, 6.00)</td>
<td>(3.00, 4.75)</td>
<td>(10.00, 5.00)</td>
</tr>
<tr>
<td>6</td>
<td>(9.75, 5.00)</td>
<td>(5.00, 9.75)</td>
<td>(10.00, 10.00)</td>
</tr>
<tr>
<td>7*</td>
<td>(58.50, 18.00)</td>
<td>(18.00, 28.50)</td>
<td>(60.00, 30.00)</td>
</tr>
</tbody>
</table>

* indicates that participants receive the stated payoffs with probability 1/6, but otherwise receive only a participation fee.

Subgame perfect Nash equilibrium: (R,r)
Results

- Player 1 trusts player 2's rationality too little.
- Player 2 mostly chooses dominant strategy.
- Payoff differences matter!

<table>
<thead>
<tr>
<th>Treatment</th>
<th># of Pairs</th>
<th>A Chose L</th>
<th>B Chose l</th>
<th>B Chose r</th>
<th>% of Secure Play by A</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>35</td>
<td>23</td>
<td>2</td>
<td>10</td>
<td>65.7%</td>
</tr>
<tr>
<td>2</td>
<td>31</td>
<td>20</td>
<td>0</td>
<td>11</td>
<td>64.5%</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td>5</td>
<td>0</td>
<td>20</td>
<td>20.0%</td>
</tr>
<tr>
<td>4</td>
<td>32</td>
<td>15</td>
<td>0</td>
<td>17</td>
<td>46.9%</td>
</tr>
<tr>
<td>5</td>
<td>21</td>
<td>18</td>
<td>0</td>
<td>3</td>
<td>85.7%</td>
</tr>
<tr>
<td>6</td>
<td>26</td>
<td>8</td>
<td>0</td>
<td>18</td>
<td>30.7%</td>
</tr>
<tr>
<td>7</td>
<td>30</td>
<td>20</td>
<td>0</td>
<td>10</td>
<td>66.7%</td>
</tr>
</tbody>
</table>
An experiment here and now

• Let’s play a simple game. No talking with one another.
• Your decision is to choose a whole number X between 0 and 100.
• The person whose number X is closest to 2/3 times the mean of all the numbers submitted will win a special prize.

• Now, please write your number on a piece of paper as follows: X = __.
• Please put your initials on the card. Do not reveal your choices to others.
Beauty Contest Game

• N Players must choose a number between 0 and 100 inclusively.
• The person(s) whose number choice $X_i$ is closest in absolute value to
  \[
p/N \sum_{i=1}^{N} X_i
  \]
  wins a fixed prize, all others earn 0
• $p$ is some value different from 1, e.g. 1/2, 2/3, 4/3
• Any $X_i > 100p$ is dominated when $p < 1$
• If $p < 1$, iterated elimination of dominated strategies leads to the choice of 0 as the unique equilibrium outcome.
• Experimental design ($p \neq 1$) allows for detection of individual steps of iterated dominance.
Beauty Contest Game

- Example of solving for the equilibrium for \( p=2/3 \).
- Winning number \( X^* \) equals \( 2/3 \) times the average \( X_{av} \)

- Step 1: \( X^* < 66.67 \). Even if \( X_{av} \) is 100 (the maximum), the best response would be to select 66.67. Therefore no one should ever choose \( X > 66.67 \)

- Step 2: Given \( X_{av} < 66.67 \), \( X^* < 44.44 \)
- Step 3: Given \( X_{av} < 44.44 \), \( X^* < 29.63 \)

- (....)

- Step ...: Given \( X_{av} < 0.00001 \), \( X^* = 0 \)
Nagel AER 1995

- $N = 15$ to $18$
- Prize to winner is 20 DM
- $p = 1/2$, $2/3$ or $4/3$
- Nash Equilibrium is 0 in first two cases, 100 in the second. It is reached by an infinite number of steps of iterated elimination of dominant strategies (fixed point solution).
Nagel's findings

Results for the treatment with $p=2/3$.
Note: The Nash Equilibrium (0) is a poor predictor of behavior.
Level-k model

- The general idea is that people differ in their level of strategic sophistication ranging from level 0 (clueless) to level $\infty$ (supersmart)
- Level 0 corresponds to non-strategic behavior where strategies are selected at random
- Level 1 players, L1, believe that all their opponents are L0 and play a best response to this belief
- Level 2 players, L2, play a best response to the belief that all their opponents are L1, etc.
Beauty Contest: Level-k

• Example of Level-k model for $p=2/3$.
• Winning number $X^*$ equals $2/3$ times the average $X_{av}$
• Level 0: randomizes uniformly on the interval between 0 and 100. Hence the average level-0 quantity ($X_0$) will be 50.
• Level 1: assumes everyone else is level 0, plays best response: $X_1=2/3*50=33.33$
• Level 2: assumes everyone else level 1, plays best response: $X_2=2/3*33.33=22.22$
• Level 3: $X_3=2/3*22.22=14.81$
• ...
The Level-k model seems to do better than Nash equilibrium.
Cognitive Hierarchy Model

• Extension of level-k model by Camerer, Ho and Chong (2004)
  • Level 0 behaves randomly (as in level-k)
    – $X_0=50$
  • Level 1 best responds to level 0 (as in level-k)
    – $X_1=50 \times \frac{2}{3} = 33.33$
  • Level 2 best responds to a mix of level 1 and level 0
    – Suppose L2 thinks 70% is L1 and 30% is L0
    – Then $X_2=\frac{2}{3} \times (0.7 \times 33.33 + 0.3 \times 50) = 25.55$
  • Level 3 best responds to a mix of level 0, 1, and 2
    – Suppose L3 thinks 35% is L1 and 15% is L0 and 50% is L2
    – Then $X_3=\frac{2}{3} \times (0.35 \times 33.33 + 0.15 \times 50 + 0.5 \times 25.55) = 21.29$
Comparison of Models

<table>
<thead>
<tr>
<th>Level/Step</th>
<th>Nash Eq.</th>
<th>Dom. Solv</th>
<th>Level-k</th>
<th>Cognitive Hierarchy</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>67</td>
<td>33</td>
<td>33</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>44</td>
<td>22</td>
<td>26 (?)</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>29</td>
<td>15</td>
<td>21 (?)</td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>∞</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>16 (?)</td>
</tr>
</tbody>
</table>

- Nash equilibrium is most parsimonious model (point prediction)
  - But the prediction is not borne out by the data
- Level-k fits/predicts better, but maybe not very realistic
- Cognitive hierarchy is the most realistic model
  - But has additional free parameter(s): the distribution of k-levels in the population.
  - And even for given level, the prediction depends on (assumed) distribution.
Another model of irrationality

• The level-k and cognitive hierarchy model assume that:
  – People have non-equilibrium beliefs (e.g., all others are level 2)
  – But they then best respond, given those beliefs.

• Another approach:
  – Assume people have equilibrium beliefs
  – But may not always best respond (exactly).

• Quantal Response Equilibrium
  – Assumes that people sometimes make small mistakes in best responding
  – Costly mistakes less likely than non costly mistakes.
  – People have correct beliefs about (noisy) behavior of others.
  – Equilibrium model: players (noisily) best respond to other players‘ (noisy) behavior.
Heterogeneity of people

- People differ in important ways, they are heterogeneous:
  - Different levels of strategic reasoning?
  - More or less likely to make mistakes?
  - Or even use different strategies altogether?
- Can be modeled and tested experimentally (see Nagel 1995)
- Problem: same outcome can often be result of different choice processes
  - E.g., in beauty contest 32 may be L0 or L2 in level k, or may be a higher level in Cognitive Hierarchy model.
How important is bounded rationality in practice?

- Bounded rationality/mistakes may cancel out on average.
  - But even then, is it not still important/interesting to investigate?
  - And: systematic deviations may affect the market equilibrium.
- Irrational individuals may learn from their mistakes.
  - But: people are not always very good at learning.
  - Learning does not always lead to equilibrium.
- Irrational individuals perform worse and are driven out of the market.
  - But: Insufficient feedback to change behavior.
  - Non-rational behavior may (ex post) be profitable.
- The influence of irrationality may depend on structure of the market.
  - Substitutability (e.g., Cournot): mistake compensated by rational agent
  - Complementarity (e.g., Bertrand): mistake amplified by rational agent
Summary: Bounded Rationality

- Rational agents always make the optimal decision.
- Real world agents are characterized by
  - Biases and use of heuristics.
  - Limited reasoning (Level-k/Cognitive Hierarchy).
  - Mistakes.
- Real world agents are heterogeneous
  - In their level of strategic reasoning (beauty contest game).
  - In the type or frequency of mistakes
  - In the types and sophistication of the strategies they use.
- Good models in behavioral economics:
  - Predict/explain behavior well in large number of settings
  - Parsimony
  - Behavioral realism.